THE ORIGIN OF THE FOUR–COLOR CONJECTURE

By Kenneth O. May *

Considering the fame and tender age of the four-color conjecture,[1] our knowledge of its origins is surprisingly vague. The well-known tradition appears to stem from W. W. R. Ball's *Mathematical Recreations and Essays,* whose first edition appeared in 1892. There it is said that "the problem was mentioned by A. F. Möbius in his lectures in 1840 . . . but it was not until Francis Guthrie communicated it to De Morgan about 1850 that attention was generally called to it . . . it is said that the fact had been familiar to practical mapmakers for a long time previously." [2] In spite of repetition by later writers, this tradition does not correspond to the facts.

In the first place there is no evidence that mapmakers were or are aware of the sufficiency of four colors. A sampling of atlases in the large collection of the Library of Congress indicates no tendency to minimize the number of colors used. Maps utilizing only four colors are rare, and those that do usually require only three. Books on cartography and the history of mapmaking do not mention the four-color property, though they often discuss various other problems relating to the coloring of maps.

If cartographers are aware of the four-color conjecture, they have certainly kept the secret well. But their lack of interest is quite understandable. Before the invention of printing it was as easy to use many as few colors. With the development of printing, the possibility of printing one color over another and of using such devices as hatching and shading provided the mapmaker with an unlimited variety of colors. Moreover, the coloring of a geographical map is quite different from the formal problem posed by mathematicians because of such desiderata as coloring colonies the same as the mother country and the reservation of certain colors for terrain features, e.g. blue for water. The four-color conjecture cannot claim either origin or application in cartography.

To support his statement about Möbius, Ball refers to an article by Baltzer, a former student of Möbius and the editor of his collected works.[3] However, as has been pointed out by H. S. M. Coxeter,[4] this article shows merely that Weiske communicated to Möbius a puzzle whose solution amounted to the claim that it is impossible to have five regions each having a common boundary with every

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[1] In nontechnical terms the four-color conjecture is usually stated as follows: Any map on a plane or the surface of a sphere can be colored with only four colors so that no two adjacent countries have the same color. Each country must consist of a single connected region, and adjacent countries are those having a boundary line (not merely a single point) in common. The conjecture has acted as a catalyst in the branch of mathematics known as combinatorial topology and is closely related to the currently fashionable field of graph theory. More than half a century of work by many (some say all) mathematicians has yielded proofs only for special cases (up to 35 countries by 1940). The consensus is that the conjecture is correct but unlikely to be proved in general. It seems destined to retain for some time the distinction of being both the simplest and most fascinating unsolved problem of mathematics.

[2] W. W. Rouse Ball, *Mathematical Recreations and Essays,* rev. H. S. M. Coxeter (London: Macmillan, 1959), p. 223.

[3] R. Baltzer, "Eine Erinnerung an Möbius und seinen Freund Weiske," *Bericht über die Verhandlungen der Sächsischen Akademie der Wissenschaften zu Leipzig. Math.-Nat. Kl.,* 1885, *37*:1–6.

[4] H. S. M. Coxeter, "The Four-Color Map Problem," *Mathematics Teacher,* 1959, *52*:283–289.

other. Baltzer witnesses that in 1840 Möbius presented this puzzle to a class and laughingly revealed its impossibility in the next lecture. But nothing was published on the problem, and its source remained unknown until Baltzer discovered Weiske's communication among the Möbius papers more than forty years later, when the four-color conjecture was already well known. Baltzer found no evidence that Möbius worked on the problem, and it is not mentioned in the collected works published in 1885–1887. Evidently Weiske's puzzle and its mention by Möbius were fruitless and had no historical link with the origin of the four-color conjecture.

The statement that attention was generally called to the problem about 1850 appears also to be not entirely accurate. Indeed the first printed reference to it appeared in 1878, when the *Proceedings of the London Mathematical Society* reported Cayley's question as to whether the conjecture had been proved.[5] Interest was immediate, and a long series of partial solutions and pseudosolutions began with the papers of Kempe [6] and Tait.[7] Cayley, Kempe, and Tait attributed the problem vaguely to Augustus De Morgan.

More precise information was supplied by the physicist Frederick Guthrie in a communication to the Royal Society of Edinburgh in 1880. He wrote,

> Some thirty years ago, when I was attending Professor De Morgan's class, my brother, Francis Guthrie, who had recently ceased to attend them (and who is now professor of Mathematics at the South African University, Cape Town), showed me the fact that the greatest necessary number of colors to be used in coloring a map so as to avoid identity of color in lineally contiguous districts is four. I should not be justified, after this lapse of time, in trying to give his proof, but the critical diagram was as in the margin. [The diagram shows four regions in mutual contact.]
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> With my brother's permission I submitted the theorem to Professor De Morgan, who expressed himself very pleased with it; accepted it as new; and as I am informed by those who subsequently attended his classes, was in the habit of acknowledging whence he had got his information.
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> If I remember rightly, the proof which my brother gave did not seem altogether satisfactory to himself; but I must refer to him those interested in the subject. I have at various intervals urged my brother to complete the theorem in three dimensions, but with little success. . . .[8]

A hitherto overlooked letter from De Morgan to Sir William Rowan Hamilton permits us to pinpoint events more precisely than could Frederick Guthrie after the passage of almost thirty years. On 23 October 1852 De Morgan wrote as follows:

> A student of mine asked me today to give him a reason for a fact which I did not know was a fact, and do not yet. He says, that if a figure be anyhow divided, and the compartments differently coloured, so that figures with any portion of common boundary *line* are differently coloured — four colours may be wanted, but no more. Query cannot a necessity for five or more be invented? As far as I see at this moment, if four *ultimate* compartments have each boundary line in common with one of the others, three of them inclose the fourth, and prevent any fifth from connexion with it. If this be true, four colours will colour any possible map, without any necessity for colour meeting colour except at a point.
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> Now, it does seem that drawing three compartments with common boundary, two and two, you cannot make a fourth take boundary from all, except inclosing one. But it is tricky work, and I am not sure of all convolutions. What do you say? And has it, if true, been noticed? My pupil says he guessed it in colouring a map of England. The more I think of it, the more evident it seems. If you retort with some very simple case which makes me out a stupid animal, I think I must do as the Sphynx did. If this rule be true, the following proposition of logic follows:
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> If *A*, *B*, *C*, *D*, be four names, of which any two might be confounded by breaking down some wall of definition, then some one of the names must be a species of some name which includes nothing external to the other three.[9]

On 26 October, Hamilton replied:

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[5] A. Cayley, "On the Colouring of Maps," *Proceedings of the London Mathematical Society*, 1878, *9*:148. A four line report of Cayley's query in the session of June 13. See also a note with the same title by Cayley in the *Proceedings of the Royal Geographical Society*, 1879, N.S. *1*:259–261.

[6] A. B. Kempe, "On the Geographical Problem of Four Colors," *American Journal of Mathematics*, 1879, *2*:193–204.

[7] P. G. Tait, "On the Colouring of Maps," *Proceedings of the Royal Society of Edinburgh*, 1880, *10*:501–503, 729.

[8] Frederick Guthrie, "Note on the Colouring of Maps," *Proc. Roy. Soc. Edinburgh*, 1880, *10*:727–728.

[9] R. P. Graves, *Life of Sir William Rowan Hamilton* (Dublin, 1889), Vol. 3, p. 423.

". . . I am not likely to attempt your 'quaternion of color' very soon." Evidently the response of others to the conjecture was equally passive. Even Francis Guthrie published nothing on it, though he lived until 1899 and produced a book and several papers on other topics.

Apparently it was on 23 October 1852 that Frederick Guthrie communicated his brother's conjecture to De Morgan, who did not bother to explain to Hamilton the indirect nature of the communication.[10] Probably De Morgan, in telling others about the problem, mentioned that it had occurred to Guthrie while coloring a map, and this gave rise to the tradition linking the conjecture with the experience of cartographers.[11]

[10] Biographical data on the Guthrie brothers supports the date indicated by De Morgan's letter. Indeed, Francis took his B.A. at University College London in 1850 and his LL.D. in 1852, whereas his younger brother Frederick was a student there in 1852 and, after a period in Germany, received his bachelor's degree in 1855.

[11] In his note of April 1879, referred to in fn. 5 above, Cayley begins: "The theorem that four colors are sufficient for any map, is mentioned somewhere by the late Professor De Morgan, who refers to it as a theorem known to map-makers." I have been unable to locate this "somewhere" and suspect that the communication was verbal. The same vague reference to De Morgan appears in a note in *Nature* (1879, *20*:275) and in Kempe's article cited in fn. 6.

On the basis of the data given here we can replace tradition by the following account of the origin of the four-color conjecture. It was not the culmination of a series of individual efforts, but flashed across the mind of Francis Guthrie, a recent mathematics graduate, while he was coloring a map of England. He attempted a proof, but considered it unsatisfactory—thus showing more critical judgment than many later workers. His brother communicated the conjecture, but not the attempted proof, to De Morgan in October 1852. The latter recognized the essentially combinatorial nature of the problem, gave it some thought, and tried without success to interest other mathematicians in attempting a solution. He communicated it to his students, giving due credit to Guthrie, and to various mathematicians, one of whom revived the problem almost thirty years later and launched it on its erratic course.

Rarely is a mathematical invention the work of a single individual, and assigning names to results is generally unjust. In this case, however, it would seem that the four-color conjecture belongs uniquely to Francis Guthrie and could fairly be called Guthrie's problem.[12]

[12] It is so called in E. Lucas, *Recréations mathématiques* (Paris, 1894).

DID THE MAYA KNOW THE METONIC CYCLE?

By David Wade Chambers *

During the past fifty years archaeologists and anthropologists have carefully documented the remarkable astronomical achievements of the ancient Maya — a civilization in which scientific investigation and religious activity were so intimately interrelated that these archaeologists have sometimes had difficulty deciding whether a given inscription offered astronomical data or religious data. Recently, historians, particularly those interested in the development of science, have begun to study archaeological researches and to incorporate a description of Maya astronomy into works of a more general character.[1]

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[1] Valuable summaries of Maya research from the perspective of the historian of science may