Homework 2, MATH 40910, Spring 2022

Due on Wednesday February 9th 2022.

1. Give a proof of the Craig interpolation theorem, Exercise 13, following the discussion in class.

2. Let F and G be formulas (involving only $\neg, \vee, \wedge$).
(i) Let F_1 result from F by replacing each literal L by $\bar{L}$, each $\wedge$ by $\vee$ and each $\vee$ by $\wedge$. Prove that $F \equiv \neg F_1$.
(ii)* Let F' result from F by replacing every $\vee$ by a $\wedge$ and every $\wedge$ by a $\vee$. Likewise for G'. Suppose $F \equiv G$. Prove that $F' \equiv G'$. (Hint: Use part (i).)

3. Using the equivalences in the Theorem on p. 15, and the substitution theorem, prove that the formula $((A \vee \neg(B \wedge A)) \wedge (C \vee (D \wedge C)))$ is equivalent to C.
(ii) Prove that every formula F is equivalent to a formula in conjunctive normal form with the same atomics (using truth tables in a similar way to what we did in class for disjunctive normal form).

4. Which of the following are true? Explain your answers, namely give a proof or counterexample.
(a) All unsatisfiable formulas are equivalent.
(b) All satisfiable formulas are equivalent.

5. (i) Let the formula F have at most the connectives $\rightarrow, \wedge, \vee$. Show that if $\mathcal{A}$ is an assignment suitable for F such that $\mathcal{A}(A) = 1$ for all atomic formulas $A \in dom(\mathcal{A})$, then $\mathcal{A}(F) = 1$.
(ii) Conclude that (for B an atomic fornula) $\neg B$ is not equivalent to any formula containing only the connectives $\rightarrow, \wedge, \vee$ and possibly containing other atomics in addition to B.